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THE FORMULA IN NINTH GRADE ALGEBRA

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Algebra is a general science. In elementary mathematics we recognize a body of mathematical material which we are accustomed to classify as arithmetic and another which we classify as algebra. Yet upon examination of the material of these two subjects we find many common problems. Does this mean that there is no sharp line of demarcation between these two subjects? In order to answer this question let us first consider some simple examples.

Let us take for our first example the problem of finding the simple interest on \$500 at 6% for a period of 2 years. The interest is obtained by multiplying \$500 by .06 and the resulting product by 2. That is

$$i = \$500 \times .06 \times 2.$$
$$i = \$60.$$

This problem is characterized by the fact that we are operating on particular numbers with a view to obtaining the interest in this particular case. This is a problem of arithmetic.

But let our attention be directed to the fact that in finding the interest in all cases in which the principal, rate, and time are given we obtain the product of these three elements. Now we are in the field of algebra.

This illustration is given in order to point out the fact that in arithmetic we are interested in particular numbers as we move toward results, while in algebra we direct our attention to the operations required to reach these results and shut out from consideration any particular characteristics of the numbers employed in the operations. In arithmetic we are interested in the combination of a set of particular numbers. In algebra we are interested in the rule for the combination of such sets. Thus arithmetic is a particular science, while algebra is a general science.

Meaning and use of the formula. Let us return to the rule for finding simple interest. It may be stated as follows: The simple interest on a sum of money at a given rate for a given number of years is equal to the product of the principal, rate, and the number of years. Now let us agree that i shall mean the phrase, "The simple interest on a sum of money at a given rate for a given number of years," and p, r, and t mean principal, rate, and number of years respectively, then the rule could be put in the form

$$i = p\,r\,t.$$

This shorthand, or shortmind,[1] form of expression of this rule is called a formula. A formula may thus be defined as a symbolic expression of a rule or principle. This symbolic mode of expression is another mark of algebra which distinguishes it from arithmetic.

Since algebra deals with generalizations expressed in symbolic language it seems that the pupil could be introduced to formal algebra quite naturally by means of the formula. In his arithmetic work he has been given various rules for the solutions of problems. He is therefore ready and will be delighted to translate these rules into formulas.

Let us consider a very simple example for illustrating the method of proceedure in making his first translations.

The number of feet in a given number of yards is equal to three times the number of yards.

It would probably be well in leading the pupil to the extreme symbolic form to first write it in an abbreviated form as follows:

No. of ft. in a given no. of yd. = 3 × no. of yd.

So far he sees abbreviations with which he is familiar. At the same time he sees an economy in the time and space required to write it, while there is a saving in mental effort in reading it.

Now let us agree that f shall mean "the number of feet in a given number of yards" and y shall mean "the number of yards." Our abbreviated form then becomes

$$f = 3 \times y.$$

Now if we make the agreement that a number placed adjacent to a letter shall mean the product of the two we can finally write

$$f = 3y.$$

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[1] See T. J. McCormack, "Why Do We Study Mathematics?"

Thus a statement of nineteen words is expressed by means of four symbols. It is much more easily read than the original statement and the relation between feet and yards can be seen at a glance.

The introduction of letters to represent mathematical phrases obviates the difficulty the pupil has in the older algebras in which he is confronted with the statement "Let x represent a number." He immediately protests that he cannot see how x can represent a number unless it be some particular number. But in the formula $f = 3\,y$ he readily sees that y stands for the phrase "number of yards" and that this number may be any one he chooses to make it.

With such an introduction to algebra considerable practice should be given in translating from ordinary language to the symbolic form, and conversely. Later I shall give examples of rules and formulas which are suitable for the ninth grade work.

The importance of the formula in mathematics can hardly be over emphasized. Formulas are found in engineering and technical journals, in works of science, in business, in a vast number of trades and professions in which a mathematical statement or investigation is required.

The importance of the formula comes from the fact that it is a great economizer of space, time, and mental effort. By means of it one can disengage his mind of all sensory experiences and can concentrate his attention wholly upon the relation or operation expressed by it. As an illustration consider the expansion of the fifth power, let us say, of the sum of two numbers, viz.,

$$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5.$$

It would require more than one hundred words to express this formula in ordinary language. Moreover it would require much more mental exort to comprehend this relation in this form than in the symbolic form.

The great advances made in mathematics date from the time of the introduction of symbols. One has only to consider such a statement as Taylor's Theorem, viz.,

$$f(x + h) = f(x) + hf'(x) + \frac{h^3}{2!}f'''(x) + \frac{h^2}{3!}f''(x) + \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

to see the necessity of a symbolism. This relation stated in ordinary language would be bewildering and probably useless for practical purposes.

Gradation of formulas. It is needless to say that formulas in a course in algebra should be arranged in the order of their complexity. However people might differ as to the position in which any particular formula should be placed. In the course which I have developed I have made the function the organizing principle. The functions which I have used are integral algebraic functions of the first and second degrees. I have arranged them in the following order.

1. $y = ax$
2. $y = ax + b$
3. $z = axy$
4. $y = ax^2$
5. $y = a(x^2 - b^2)$
6. $y = a(x + b)^2$
7. $y = ax^2 + bx + c$

Now a formula may be looked upon as a relation between variables. If one of the variables in a formula is expressed in terms of the others it is said to be a function of these variables. Thus in all of the relations above, excepting (3), y is a function of x; while in (3) z is a function of x and y. If, therefore, a formula can be put in either of the forms listed it should be placed in the course under the function to which it belongs.

Formulas should be interesting. No formula should be given a pupil unless it concerns something of interest to him. Whenever convenient it should be derived by him. Thus the formula,

$$F = \tfrac{9}{5} C + 32,$$

has much more interest and meaning for the pupil who has actually worked out the relation between the two thermometer scales, who has expressed this relation in good English, and who finally, has translated his statement into this shorthand form.

There is a vast amount of material which may be used in formula construction. The rules of arithmetic may be drawn upon. Experimental geometry offers a rich field. Here we can find formulas on perimeters, similar figures, and areas. Then there are many mechanical and physical laws within the easy comprehension of the pupil. Among these are uniform motion, forces on an inclined plane, motion on an inclined plane, motion under constant acceleration, vibration of the simple pendulum, and distance traversed by a body moving under constant acceleration. Many business rules may also be expressed by formulas falling within the range of the ninth grade work.

Formulation and translation. No attempt should be made by the pupil to construct a formula until the situation he wishes to express by it has been carefully analyzed and formulated in words. As a simple example consider the construction of the formula for the area of a rectangle. After the pupil has learned that the area of a rectangle means the number of square units that will cover it, let him be required to find the area of a rectangle, 8 by 11 inches, let us say, which has been divided into square inches without counting the individual units. He soon discovers that this may be done by counting the number of squares in a row and multiplying the result by the number of rows. He is then ready, so far as the case of integral dimensions is concerned, and should be required to state that the area of a rectangle is equal to the product of the base and altitude. Now he may translate this into the formula,

$$A = ab$$

Throughout the course the pupil should have much practice in the formulation of mathematical statements in words and the translation of these statements into formulas. Conversely, he should be required to translate formulas into ordinary language and to see that an elegant and precise rendering has been made.

Some Examples of Material to be Used. I shall now add some typical examples which I am accustomed to give my classes. They are adapted to ninth grade work and are arranged in the order in which they are presented.

1. Write a formula for finding the perimeter of a square when the number of feet in the length of a side are given (p, s).

Before writing this formula the pupil should be required to state the rule in words. The notation (p, s) signifies that he is to let p mean "the number of feet in the perimeter" and s, "the number of feet in the length of the side." The advantage gained by having pupils use the same notation for purposes of comparison is obvious.

2. Motion is frequently transmitted from one piece of machinery to another by means of a belt. The safe working strength, expressed in pounds, of a belt per inch of width is equal to 300 times the belt expressed in inches. Express this statement by means of a formula (s, t).

3. A rule used by some dairymen in feeding grain to dairy cows is as follows: To determine the number of pounds of grain to feed a dairy cow per day divide the number of pounds of milk produced by her by 3.5. Write the formula (g, m).

4. A train travels 40 miles per hour. Let d mean distance in miles, and t, time in hours, then

$$d = 40t.$$

Express this formula in words.

5. A formula which will give the time between any two consecutive hours when the hands of a clock are together is

$$m = \tfrac{60}{11} h$$

where m is the number of minutes past the hour and h is the number of hours. Translate.

6. With an inclined plane, car, and spring balances from the physics' laboratory it is easy to have the pupils see that

$$\frac{p}{w} = \frac{h}{l}$$

where p and w mean power and weight, and h and l mean height and length of the plane, and provided the power acts in a direction parallel to the plane.

From this relation pupils can find

$$p = w \cdot \frac{h}{\ }$$

or $$p = w sin i$$

where i means the angle of inclination.

Of course this formula presupposes familiarity on the part of the pupil with the trigonometric ratios.

8. It is not difficult to lead a class to see that velocity on an inclined plane is given by the formula

$$v = 32tsini.$$

The formulas dealt with above are of the form

$$y = ax.$$

A vast amount of material giving rise to relations of this form is available. The formulas which follow immediately fall under the more general form

$$y = ax + b.$$

9. A recipe for making coffee is the following. Allow one tablespoonful for each cup and one for the pot. Translate this rule into a formula (t, c).

10. I bought a set of books for which I was to pay $18. I agreed to pay for them at the rate of $1 per month. Write a formula expressing the amount still owed at the end of a given number of months (a, t).

11. A certain society charges $10 for the initiation and $5 per year for dues. Write a formula for computing the amount paid in by a member after any number of years (a, t).

12. The Chicago Telephone Company's rate for a four-party line is as follows: A guarantee of 5 cents per day, *i.e.*, $1.50 per month, plus 4 cents for each outgoing message above 30. Write a formula for finding the cost for any number of messages (c, m).

13. Given a table of rates of postage for the various zones the pupil can easily construct formulas giving the postage for each zone on any number of pounds. These formulas are of the form

$$p = f + c(w - 1).$$

14. As a train increases its speed the resistance to its motion, due to the air and other things, increases. The following formula has been suggested for computing this resistance, viz.,

$$r = 2 + \frac{s}{4}$$

where r means the number of pounds of resistance per ton of load and s, the speed in miles per hour.

Translate this formula into a rule for computing resistance.

15. The formula for computing the postage on a parcel directed to a local point is

$$p_0 = .05 + .01\,\frac{w-1}{2},$$

where p_0 is expressed in dollars and w in pounds. Translate.

16. The swimming tank in Hyde Park High School has a capacity of 54,000 gallons. It can be filled at the rate 1,500 gallons per hour. At a certain instant while it is being filled it contains 22,500 gallons. Write a formula by which the number of gallons in the tank may be computed for any time before or after this instant (g, t).

The formula is

$$g = 22{,}500 + 1{,}500t.$$

t may take both positive and negative values. g is always positive.

17. A dairyman, who retailed his milk, derived a formula by which he could compute the approximate annual profit on a cow if he knew the average number of pounds of milk she gave per day. The formula was

$$p = 15m - 135,$$

where p is the profit in dollars and m is the average number of pounds of milk per day

Notice that p may take both positive and negative values while m is always positive.

18. When weights of 2 ounces, 6 ounces, 8 ounces, and 12 ounces are suspended from a rubber cord, its lengths are found to be 7 inches, 9 inches, 10 inches, and 12 inches, respectively. Write a formula for computing the length for any other weight (l, w).

19. A railway station, B, is 190 miles east of a station, A. A train running on a road passing through A and B is leaving B and going east. Distance east of A and velocity east are considered positive. If the train runs at an average rate of 40 miles per hour, what will be the formula giving the distance, d, from A at the time, t, time being counted from the instant that the train leaves B.

The formula is

$$d = 190 + 40t.$$

This is an excellent formula for giving practice in the interpretation of directed numbers.

20. Write a formula for computing the n^{th} term of the arithmetic progression,

$$\cdots\cdots\cdots\ 2,\ 7,\ 12,\ 17,\ \cdots\cdots\cdots$$

starting from 12.

21. Two trains are now d_0 miles apart and are traveling on a straight road at the rates of r_1 and r_2 miles per hour. Write a formula giving their distance, d, apart at any time, t.

The formula may be written as

$$d = d_0 + (r_2 - r_1)t$$

It has a broad application since it can be used in expressing the relation between objects which are changing uniformly either in the same or opposite senses. It offers an excellent opportunity for practice in the interpretation of directed numbers.

22. A boy rolls a ball up a smooth incline with velocity of v_0 feet per second. Its velocity decreases uniformly at the rate of a feet per second. Write a formula giving the velocity of the ball at any time (v, t).

23. A sea-coast town, Q, bears N25°E from another coast town, P. A ship is sailing from P to Q at the rate of 18 miles per hour and is now 4 miles directly east of a coast town, T. Let distance north and east and rates in a northerly direction be positive. Write a formula giving the departure of the ship from T at other times (d, t).

The formula is

$$d = 4 + 18t\sin 25°.$$

Formulas of this type give practice both in the use of trigonometric ratios and in interpreting directed numbers.

24. A point moves so that its distance from the x-axis is 3 increased by twice its distance from the y-axis. A second point moves so that its distance from the x-axis is 6 diminished by its distance from the y-axis. Find the co-ordinates of the point of intersection of their paths.

25. Two rubber cords hang side by side suspended from a horizontal support. One is 18 inches long and is stretched $1\frac{3}{8}$ inches by each additional ounce hung at its end. The second

is 21 inches long and is stretched 7/16 inch by each additional ounce. What weight will stretch them to the same length and what will the length be?

26. The pupil will have little difficulty in deriving the formulas for the areas of parallelograms, triangles, and trapezoids, viz.,

$$A = bh,\ A = \tfrac{1}{2}bh,\ A = \tfrac{1}{2}h(a + b).$$

27. If two adjacent sides of a parallelogram are a and b and the included angle is α, then

$$A = ab \sin \alpha.$$

Similarly for the triangle

$$A = \tfrac{1}{2}ab \sin \alpha.$$

28. From the definition of square formulate a rule for finding its area and express it as a formula (A, s).

29. The load that may be safely carried by an iron chain is given by the formula

$$l = 7.11d^2,$$

where l is the load in tons and d is the diameter of a link expressed in inches. Translate.

30. The time of vibration of a simple pendulum is given by the formula

$$t = .554\sqrt{l},$$

where l is the length of the pendulum in feet and t is the time in seconds. Translate.

31. If the formula giving the velocity of a falling body is

$$v = 32t,$$

show that the distance, s, through which it falls in t seconds is

$$s = 16t^2.$$

The derivation of this formula requires the use of the formula for the sum of an arithmetic series.

32. From the definition of annulus show that its area is given by the formula

$$A = \pi(r_1^2 - r_2^2)$$

33. In a ''potato'' race the potatoes were placed in a straight line at equal intervals of 10 feet. A basket, in which a contestant was to deposit each potato in turn, was placed at a distance of 15 feet from the first potato and in the line of potatoes. Show that the total distance traveled by a contestant is

$$d = 10n^2 + 20n,$$

where n is the number of potatoes.

34. If a body with an initial velocity v_o moves under constant acceleration, a, so that its velocity is

$$v = v_o + at,$$

show that the distance passed over during time, t, is

$$s = v_ot + \frac{1}{2}at^2.$$

Since the letters in this formula represent directed numbers, it offers an opportunity for more practice in their interpretation.

I wish to turn aside here to add a remark in regard to the use of simple material from mechanics. There seems to be a disinclination on the part of some mathematics teachers to use such material on the ground that it takes too much time for the demonstration of mechanical principles, and anyhow they belong to the department of physics.

Of course on the same ground one could object to the use of geometrical material since its takes time to get geometrical principles before the pupils, and anyhow they belong to the course in geometry. On the same ground one might object to introducing any material for the purpose of making applications.

The answer to these objections is obvious. The majority of ninth grade pupils will never take a course in physics. The simple mechanical laws do not require much time for their illustration. Moreover, mathematics is just as much concerned with mechanical laws as it is with geometrical laws or any kind of laws. Our high school mathematics in the past has been formal and, so far as the average pupil is concerned, sterile. Our pupils have asked us to give them a *raison d'etre* of algebra and the only justification we could give was that it possibly trained their minds and, if they continued work in mathematics, they would need it in later courses.

These two answers are really identical. For, if a pupil is given nothing but formal mathematics, his mind will be trained for that sort of mathematics and that only. If we wish algebra to function in the lives of our pupils we must see to it that it has many points of contact with their experiences. There is no transfer of mental training from algebra unless the teacher sees to it that the pupil identifies its laws with the laws of nature and society about him.[1]

Evaluation of Formulas. If a formula is written in the explicit form it may be considered as a general solution. To obtain a particular solution we need only substitute the given numerical values of the letters and then simplify. Since the necessity for the evaluation of formulas occurs very frequently not only in mathematics but also in all sorts of scientific and technical work, it is highly desirable that the pupil should be given a great deal of practice in substitution from the beginning of the course.

If a number of particular solutions for a letter in a formula are required it is probably desirable to solve the formula beforehand for this letter and then substitute the given values of the other letters. However, if one meets a problem in which it is required to find only one numerical value of a letter which is not expressed explicitly in terms of the others, it is better to substitute the numerical values of the other leters in the formula and then solve the resulting equation for this letter. Thus a large amount of legitimate equation material may be obtained in this way.

Moreover, in general, it is better that a pupil become familiar with but one form of a formula. For example, it is better that the formula for uniform motion should be learned in the form $d = rt$ rather than in all three forms. Or, that the formula,

$$s = v_0t + \tfrac{1}{2}at^2,$$

should be known in this form rather than in a number of forms in which it might be expressed.

In solving a problem involving a formula the pupil should be required to write the formula first. On the part of the pupil this guarantees that he think out the plan of his solution at the start. On the part of the teacher it means a saving of time and energy in discovering the plan of the pupil. After the formula has been written the known value of the letters should be set down. These values should then be substituted in the formula and the resulting equation solved.

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[1] C. H. Judd, "Psychology of High School Subjects." Especially the chapter on "Generalized Experience."

Let us consider some examples of equations which arise from substitution in formulas and the form in which the work should be written.

Example 1. What time will be required for a ball to attain a velocity of 22.3 feet per second in rolling down a smooth incline which has an inclination of 6° ?

Solution.

$$\begin{aligned} v &= 32.2t\sin i \\ v &= 22.3 \\ \sin 6^\circ &= .105 \\ \therefore 22.3 &= 32.2 \times .105t \\ 22.3 &= 3.38t \\ t &= 6.63 \end{aligned}$$

Computation:

$$\begin{array}{r} 32.2 \\ .105 \\ \hline 3.22 \\ 16 \\ \hline 3.38 \end{array} \qquad \begin{array}{r|l} 22.30 & 3.38 \\ \cline{2-2} 20\ 16 & 6.63 \\ \cline{1-1} 2\ 14 & \\ 2\ 03 & \\ \cline{1-1} 11 & \\ 10 & \\ \cline{1-1} \end{array}$$

Example 2. Change —58° F. into degrees C.

Solution:

$$\begin{aligned} F &= \tfrac{9}{5}C + 32 \\ F &= -58 \\ \therefore -58 &= \tfrac{9}{5}C + 32 \\ -90 &= \tfrac{9}{5}C \\ -450 &= 9C \\ C &= -50 \end{aligned}$$

Example 3. An automobile is traveling at the rate of v_0 miles per hour when it begins to change its velocity a miles per hour every minute. If the velocity is now 27 miles per hour, the original velocity 15 miles per hour, the change in velocity $+3$ miles per hour every minute, how many minutes has it been changing its velocity ?

Solution.

$$\begin{aligned} v &= v_o + at \\ v &= 27 \\ v_o &= 15 \\ a &= 3 \\ \therefore 27 &= 15 + 3t \\ t &= 4 \end{aligned}$$

Example 4. A rifle was fired horizontally from the top of a cliff 100 feet above the level of a lake. The bullet had a horizontal velocity of 800 feet per second. How far from the foot of the cliff did it strike the water?

Solution.

$$\begin{aligned} s &= 16t^2 \\ s &= 100 \\ 100 &= 16t^2 \\ t &= 5/2 \end{aligned}$$

$$\begin{aligned} d &= rt \\ r &= 800 \\ t &= 5/2 \\ \mathrm{d} &= \tfrac{5}{2} \times 800 \\ d &= 2000 \end{aligned}$$

Solving a Formula. Since it is sometimes desirable to express one of the letters of a formula explicitly in terms of the others and since, also, a change in the form of a formula may give additional information, the pupil should be given practice in solving formulas for any letter. If the steps in the solution of a numerical case are noted, it becomes a simple matter to write down the corresponding general solution. Thus if the pupil is required to solve the formula,

$$F = \tfrac{9}{5}C + 32$$

for C, let him first find the value of C let us say for $F = 68$ without making any combinations. Thus:

$$\begin{aligned} 68 &= \tfrac{9}{5}C + 32 \\ 68 - 32 &= \tfrac{9}{5}C \\ 5(68 - 32) &= 9C(68 - 32) \\ \mathrm{C} &= \tfrac{9}{5}(68 - 32) \end{aligned}$$

Now it becomes evident to the pupil that 68 may be replaced by F thus obtaining

$$C = \tfrac{5}{9}(F - 32).$$

After some practice in this proceedure he should be required to solve for letters as if they were numbers.